

SOLUTION OF THE EQUATION OF CONVECTIVE - DIFFUSIVE  
TRANSFER OF AN ADMIXTURE INSIDE A CYLINDER IN THE  
PRESENCE OF FREE GAS EXCHANGE BETWEEN THE FLOW  
AND WALL

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The results are presented of a theoretical and experimental study of convective-diffusive transfer of an admixture inside a cylinder for free gas exchange between the flow and wall when  $Re \geq 3.3 \cdot 10^4$ .

When studying certain problems of the aerogas dynamics of mines it is necessary to consider the transfer of sorbed admixtures with the ventilation flow along the mines. Such a case occurs, for example, during movement of inert gases in a working, driven along a coal seam, when the gases are sorbed by the exposed surfaces of the seam.

It is necessary to consider the movement of inert gases along mines in the case of the release of gases into fire zones being isolated to prevent an explosion.

We will consider below the problems of the dynamics of the concentrations of sorbed inert gases in a turbulent ventilation flow for  $Re \geq 3.3 \cdot 10^4$ .

The following assumptions are made in formulating the problem: 1) there is no accumulation of the sorbed admixture in the seam, which holds in practice for short time intervals; 2) the mine is represented as a semiinfinite cylinder; 3) the concentration of the admixture is distributed uniformly in the initial section.

Since processes of the movement of gases represent typical mass-transfer processes and in the given problem we are considering the transfer of an admixture, then taking account of the assumptions made, the gas flow through the mine workings can be described by the following equation [1]

$$\psi \left[ \frac{\partial^2 c}{\partial x^2} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial c}{\partial r} \right) \right] = u \frac{\partial c}{\partial x} + \frac{\partial c}{\partial t} \quad (1)$$

with initial and boundary conditions

$$\begin{aligned} c(x, r, 0) &= 0; \quad c(0, r, t) = f(t); \\ c(\infty, r, t) &= 0; \quad \left. \frac{\partial c}{\partial r} \right|_{r=a} + \alpha c \Big|_{r=a} = 0. \end{aligned} \quad (2)$$

The last of the boundary conditions reflects absorption of the admixture by the walls of the mine with a constant gradient, i.e., directly proportional to the concentration.

Performing in Eq. (1) and in boundary conditions (2) the substitutions  $x = \alpha \xi$ ,  $r = a\rho$ ,  $t = a^2\tau/\psi$  and using the Laplace transform with respect to variable  $\tau$ , we obtain

$$\frac{\partial^2 G}{\partial \xi^2} + \frac{1}{\rho} \frac{\partial}{\partial \rho} \left( \rho \frac{\partial G}{\partial \rho} \right) = v \frac{\partial G}{\partial \xi} + \rho G, \quad (3)$$

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$$G(0, \rho, \tau) = \Phi(\rho), \quad G(\infty, \rho, \tau) = 0, \quad \frac{\partial G}{\partial \rho} + AG = 0, \quad (4)$$

where  $A = \varphi\alpha$ , and  $G$  and  $\Phi(\rho)$  are transforms of functions  $c$  and  $f(t)$ , respectively.

Applying to (3) the integral transformation  $\int_0^{\tau} G\rho J_0(\lambda\rho)d\rho = q$  and using the last of boundary conditions (4), we obtain the equation

$$\frac{\partial^2 q}{\partial \xi^2} + (\lambda^2 + \rho)q - v \frac{\partial q}{\partial \xi} = 0, \quad (5)$$

the solution of which taking account of the condition  $G(\infty, \rho, \tau) = 0$  is

$$q = c_1 \exp \left[ \left( \frac{v}{2} - \sqrt{\frac{v^2}{4} + \lambda^2 + \rho} \right) \xi \right]. \quad (6)$$

The solution for  $G$  can be represented by the series

$$G = \sum_{n=1}^{\infty} A_n J_0(\lambda_n \rho) \exp \left[ \left( \frac{v}{2} - \sqrt{\frac{v^2}{4} + \lambda_n^2 + \rho} \right) \xi \right]. \quad (7)$$

Finding  $A_n$  with respect to the first of conditions (4) and then passing to the original and returning to the old variables, we obtain

$$c = \sum_{n=1}^{\infty} B_n \int_0^{\sqrt{t}} \frac{F(t - \eta^2)}{\eta^2} \exp \left[ -\frac{1}{4} \psi \left( u\eta - \frac{x}{\eta} \right)^2 - \frac{\sqrt{\psi}}{a} \lambda_n^2 \eta \right] d\eta, \quad (8)$$

where

$$B_n = \frac{2x}{\sqrt{\psi\pi}} \cdot \frac{J_0\left(\lambda_n \frac{r}{a}\right) J_1(\lambda_n)}{\lambda_n [J_0^2(\lambda_n) + J_1^2(\lambda_n)]};$$

the  $\lambda_n$  are the roots of the equation  $AJ_0(\lambda) + J_0'(\lambda) = 0$ .

Averaging  $c$  over  $r$ , we obtain

$$c_{av} = \frac{4x}{\sqrt{\pi\psi}} \sum_{n=1}^{\infty} \frac{J_1(\lambda_n)}{\lambda_n^2 [J_0^2(\lambda_n) + J_1^2(\lambda_n)]} \int_0^{\sqrt{t}} \frac{F(t - \eta^2)}{\eta^2} \exp \left[ -\frac{1}{4} \psi \left( u\eta - \frac{x}{\eta} \right)^2 - \frac{\sqrt{\psi}}{a} \lambda_n^2 \eta \right] d\eta. \quad (9)$$

The results of calculations by Eq. (9) were checked by comparison with field observations directly under mine conditions.

The experimental conditions were as follows. In the initial section ( $x = 0$ ), carbon dioxide was released at a certain rate into the ventilation flow passing along the mine. Samples of the carbon dioxide-air mixture were collected in evacuated bottles at various distances from the initial section at various times. The samples were collected simultaneously at several points of the section, and, after analysis the average concentrations over the section were calculated.

In one of the experiments, the carbon dioxide was released so that its average concentrations in the initial section varied according to the function  $f(t) = 0.0237t^4 \exp[-0.1335t]$ . The average velocity of the gas-air mixture over the section was 0.1 m/sec. At a distance of 180 m from the initial section we took samples and then calculated the average  $\text{CO}_2$  concentration over the section. The same concentrations were calculated by Eq. (9) on the Ural-4 computer according to a program using the Gauss method for calculation (the number of nodes was 40).

The theoretical curve of the change of carbon dioxide concentration and the experimental values are shown in Fig. 1. As follows from Fig. 1, the agreement of the measured values of the concentration with the calculated values is satisfactory.

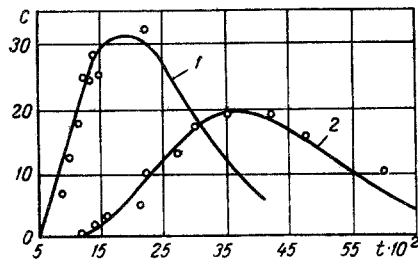


Fig. 1

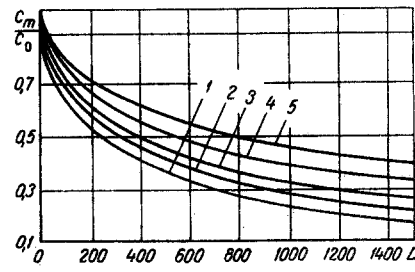


Fig. 2

Fig. 1. Theoretical curves and results of observations of the change of carbon dioxide concentration, %, in time, sec, in the initial section (1) and at a distance of 180 m from it (2).

Fig. 2. Curves of the change of the relative carbon dioxide concentrations along the path of its movement through the mine at flow velocities: 1) 0.15; 2) 0.30; 3) 0.60; 4) 2.0; 5) 5.0 m/sec.

For practical releases of carbon dioxide into fire zones, the change of CO<sub>2</sub> concentration in the initial section occurs most often according to relations described by the expression

$$f(t) = k[1 - \exp(-\alpha t)]. \quad (10)$$

Substituting (10) into (9) and calculating the values of  $c$  for different  $x$  and  $t$  with other constant values of the parameters, we obtain the change of CO<sub>2</sub> concentrations according to curves similar in appearance to curve (10). In this case, however, the maximum concentrations of the gas in the wave decrease as it moves through the mine.

Figure 2 shows the curves of the relation between the relative gas concentration and the distance from the place of its release for different flow velocities  $u$  and constant values of  $\alpha$  and  $\psi$ . Here the relative concentration means the ratio of the maximum concentration at a given distance from the initial section to the concentration in the initial section. As we see from Fig. 2, the gas concentrations decrease with an increase of the distance from the place of releasing the gas. The gas concentrations increase with an increase of the flow velocity for a fixed value of  $x$ .

When releasing carbon dioxide into fire zones being isolated for the purpose of preventing an explosion it is usually necessary to fill the zones with gas to concentrations of the order of 30%, thereby reducing the oxygen content to 15%. We can conclude from the graph presented that it is possible to achieve such a CO<sub>2</sub> concentration, for example, at a flow velocity of 0.15 m/sec at distances up to 700 m only and at a flow velocity of 0.6 m/sec at distances up to 1200 m.

These relations for the change in concentration of the gas over its travel path were checked in experiments under mine conditions and good agreement was found between the theoretical and experimental values.

#### NOTATION

$c$	is the concentration of admixture, %;
$r, x$	are the space coordinates, m;
$t$	is the time coordinate, sec;
$a$	is the radius of cylinder, m;
$\psi$	is the coefficient of turbulent mixing, m <sup>2</sup> /sec;
$u$	is the flow velocity, m/sec;
$f(t)$	is the function of the change of concentration in the initial ( $x = 0$ ) section, %;
$\alpha = A/\varphi$ ;	
$A$	is the sorption of admixture by unit surface area, m <sup>3</sup> /sec/m <sup>2</sup> ;
$\varphi$	is the coefficient of diffusion between flow and wall, m <sup>2</sup> /sec;
$\xi = x/a$ ,	
$\rho = r/a$ ,	
$\tau = t\psi/a$	are the dimensionless coordinates;
Re	is the Reynolds number.

#### LITERATURE CITED

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